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The Language and Basic Phenomena of Nonlinear Dynamics in Vocal Fold Vibration

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Journal of Singing, January/February 2020
Volume 76, No. 3, pp. 295–297

Dynamics is the branch of mechanics concerned with the motion of bodies under the action of forces. When the resulting motion is not proportional to the applied forces, the process is said to be nonlinear. The dynamics of vocal fold vibration involve many nonlinear actions as aerodynamic energy is converted to acoustic energy. To describe these processes adequately, the language and fundamental concepts of nonlinear dynamics are useful. First, we must understand the definition of nonlinearity clearly. A device, a machine, or a full system of devices or machines coupled together, behaves linearly if the output is in direct proportion to the input. Consider bicycle dynamics as an example. The rotation of the wheels (the output) is directly proportional to the rotation of the pedals (the input). With wheel sprockets and a chain coupling the pedals to the rear wheel, circular motion at the input results in circular motion at the output. The system of dynamical parts from the pedals to the wheels is linear.

When the output is not proportional to the input, the system is said to be nonlinear. Vocal fold vibration is a perfect example. The input is a steady lung pressure and a steady flow of air, but the output is an oscillatory (back and forth) movement of vocal fold tissues. This nonlinear phenomenon has only been fully accepted in voice science for about 70 years. Prior to 1950, a neurochronaxic theory of vocal fold vibration was deeply under discussion, with the belief that periodic (oscillatory) nerve impulses to the thyroarytenoid muscle resulted in synchronized periodic vocal fold vibration. While the neurochronaxic theory is no longer accepted for vocal fold vibration, it is still alive for theories of vocal tremor and vibrato.

Self-sustained oscillation in a nonlinear system can have multiple quasi-stable states, also known as *attractors*. In its continuous back and forth motion, the system can be *attracted* to any one of these states, but not necessarily permanently. Only quasi-stability exists because a small change in the input (lung pressure, muscle activation, etc.) or the internal parameters (stiffness, viscosity, adduction, etc.) can quickly flip the quasi-stable state to a new *attractor*. The output associated with this new *attractor* can be very different from the previous one. The *attractor* states are said to be *bifurcated*. For example, periodic vibration can change to *chaotic* vibration

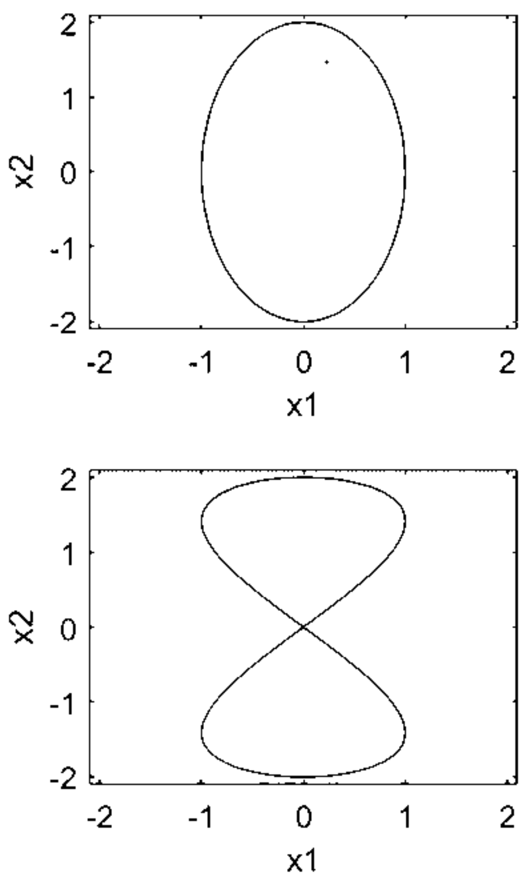

Figure 1. Simple attractors in phase space. Top: *period-1* oscillation. Bottom: *period-2* oscillation.

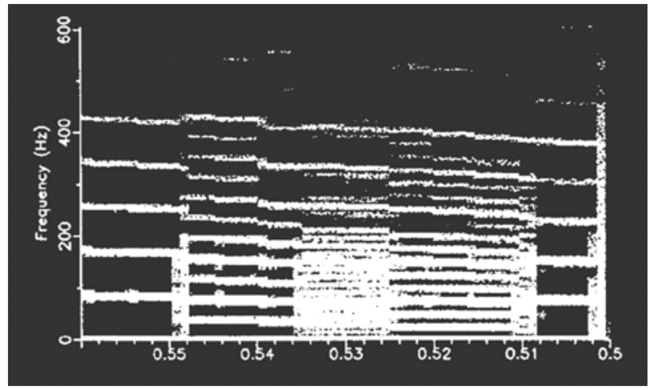

Figure 2. Spectrogram of the acoustic output from a two-mass model of the vocal folds with left-right asymmetry. A nondimensional (ratio) parameter for symmetry in vocal fold tension and mass was varied in small increments.

(no detectable period). Perceptually, a tonal sound suddenly becomes a rough (atonal) sound. The change is often unpredictable.

Attractors are generally quantified in *phase space.* Two variables are plotted against each other (on the x and y axis) that are generally not in phase (not proportional to each other). The phase space figure can look very simple or very complicated, depending on the nature of the attractors. Two frequently observed simple attractors in vocal fold vibration are the *period-1* attractor and the *period-2* attractor (Figure 1). As the names suggest, the *period-1* attractor has one fundamental period, and hence one fundamental frequency. Vocal fold displacement x1 plotted against vocal fold velocity x2 constitute a good pair of plotting variables because they are naturally out of phase in simple oscillation. They produce the kind of elliptical attractor shown on top of Figure 1. As time elapses, the elliptical path is traversed over and over again. On the bottom, the vibration is also periodic, but a *period-2* motion is also present. Note that there are two back and forth movements vertically for every one back and forth movement horizontally. Again, as time elapses, the figure-8 path is followed repeatedly.

Each of these motions can generate a series of harmonic frequencies in the acoustic output. The harmonic frequencies come from further nonlinearities in the vocal system, primarily the nonlinearities from vocal fold collision and the nonlinearities from aero-acoustic interaction with the vocal tract. Hence, there can be multiple *harmonic series*, and there is theoretically no limit to the number of harmonic series that can be produced. In nonlinear dynamic terminology, *period-n bifurcations* can exist, where *n* is any positive integer. The reader is encouraged to go online, search for "attractors in phase space" and observe the many interesting attractors described with phase plots,

The spectrogram is a useful tool for qualitative assessment of nonlinear phenomena in vocal fold vibration. It shows how periodicity can change over time. Figure 2 shows a spectrogram with multiple *bifurcations*. It was produced with an asymmetric two-mass model of the vocal folds. Note that vibration begins with a *period-1* segment (a single harmonic series), followed by a *period-2* segment (an additional subharmonic series), followed by a *chaotic* segment, followed by a *period-3* segment, and finally returning back to a *period-1* segment. Left-right asymmetry in vocal fold stiffness and mass was quantified by an internal parameter that could

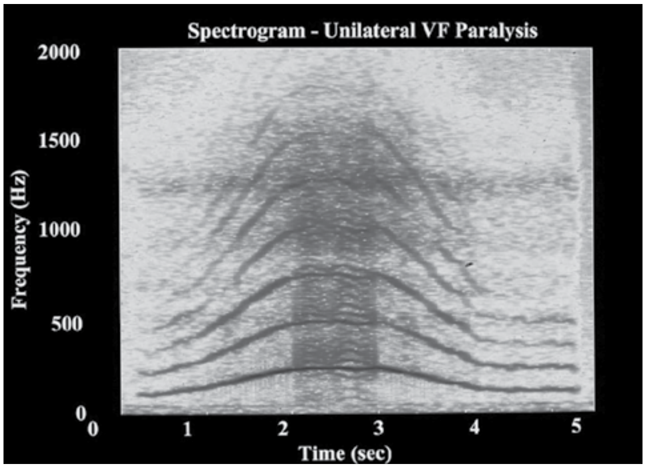

Figure 3. Spectrogram of a female with unilateral vocal fold paralysis producing a pitch glide. A bifurcation in vocal fold vibration occurs at the top of the glide.

vary from 1.0 (perfect symmetry) to near zero (much asymmetry). In the experiment, it was changed every 400 milli-seconds in 0.01 steps from 0.55 to 0.50. Note that small changes in the asymmetry parameter (a few percent) can produce large changes in the output. This is the nature of nonlinear dynamics.

Figure 3 shows a spectrogram of a female patient with unilateral vocal fold paralysis executing a pitch glide. Note the sudden bifurcation after two seconds from *period-1* oscillation with a harmonic series to multiple periods (*period-n* and perhaps some chaotic vibration). These periods also exhibit harmonic series in the measured acoustic output. At three seconds, the system returns to the original *period-1* vibration on the downward glide.

The take-home message for singing teachers is that vocal instabilities and sudden gross changes in vocal quality are sometimes not easy to control with simple input changes at the muscular level. The physical plant organizes and disorganizes itself in complex ways. With well constructed and repeated exercises, however, transitions can be made smoother and more predictably. In effect, a nonlinearity in the system can in part be negated by a nonlinearity in the control system.

REFERENCES

Multiple figures displayed online on the topic of "attractors in phase space."