Mathematics Homework

1. Simplify $x = (56 + 18 \times 7) - 22(0.5 * 17)$.

2. $\lambda = \frac{s}{f}$ Solve for f where $\lambda = 0.78$ and $s = 340$.

3. Simplify $x = 10^{-0.28}$.

4. Simplify $x = \log_{10} 0.89$.

5. Simplify $x = \text{antilog}_{10}\, 1.68$.

6. Convert to scientific notation 54,300,000.

7. Convert to scientific notation 862.57×10^{-5}.

8. Convert 78° into radians.

9. Calculate the sine of the angle $\theta = 54°$.

10. Convert Cartesian coordinates (51, 17) to polar coordinates.

Chapter 1: Mathematics

Emanuel & Letowski (2009)

Math topics for hearing science

- Arithmetic
- Algebra
- Geometry
- Trigonometry

Equations

An <u>equation</u> is a statement asserting the equality of two quantities.

$$7 \times 3 = [(z + 3) \times (2 - 7)]$$

Solving equations

<u>Solving an equation</u> means rewriting it so that one side of the equation is an unknown variable and the other side is as simple as possible.

Steps to solving equations

- Substitution
- Simplification
- Solving for x

Substitution

<u>Substitution</u> is the act of replacing a variable with something else.

  - A <u>variable</u> is a letter or symbol representing an unknown number

Substitution (cont.)

What are the variables in this equation?

$$\lambda = c/f$$

λ, c, and f are all variables

Substitution (cont.)

If we know that f = 1000 and c = 340, what would the equation look like after substitution?

$$\lambda = c/f$$

$$\lambda = 340/1000$$

And then after simplification:

$$\lambda = .34$$

Simplification

- Simplification is a step-by-step process for making an equation simpler.
- Simplification follows the Order of Operations.

Order of Operations (PEMDAS)

1. Operations in **P**arentheses
2. **E**xponents (logs)
3. **M**ultiplication and **D**ivision
4. **A**ddition and **S**ubtraction

Example

$$x = 45 + 7 \times (45 + 2) + 10^2$$

$$x = 45 + 7 \times \underline{(45 + 2)} + 10^2$$

$$\downarrow$$

$$x = 45 + 7 \times 47 + 10^2$$

Example (cont.)

$$x = 45 + 7 \times 47 + \underline{10^2}$$

$$\downarrow$$

$$x = 45 + \underline{7 \times 47} + 100$$

$$\downarrow$$

$$x = 45 + 329 + 100$$

Example (cont.)

$x = \underline{45 + 329 + 100}$

↓

$x = 474$

Solving for x

Solving for x means manipulating an equation to isolate x on one side.

- x may be represented by other letters
- For example: d, t, v, θ, λ, etc.

Solving for x (cont.)

Whatever you do to one side of an equation you must do to the other side of the equation

Example

$x + 2 = 14$

$x + 2 \underline{\mathbf{- 2}} = 14 \underline{\mathbf{- 2}}$

$x = 12$

Solving for x (cont.)

To solve for x when there is more than one mathematical step, you must use the reverse Order of Operations.

Solving for x (cont.)

Instead of PEMDAS use SADMEP.

- **S**ubtraction & **A**ddition
- **D**ivision & **M**ultiplication
- **E**xponentiation
- **P**arentheses

Example with substitution, simplification, and solving for x

$x = 59 - 4 + 6y + 20 \times 10^3$

Solve for y where x = 1.3

Example (cont.)

$x = 59 - 4 + 6y + 20 \times 10^3$

$1.3 = 59 - 4 + 6y + 20 \times \underline{10^3}$

$1.3 = 59 - 4 + 6y + \underline{20 \times 1000}$

$1.3 = 59 - 4 + 6y + 20{,}000$

Example (cont.)

$1.3 = \underline{59 - 4} + 6y \underline{+ 20{,}000}$

$1.3 = 6y + 20{,}055$

$1.3 \; \underline{\mathbf{- 20{,}055}} = 6y + 20{,}055 \; \underline{\mathbf{- 20{,}055}}$

$-20{,}053.7 = 6y$

Example (cont.)

$-20{,}053.7 = 6y$

$-20{,}053.7 \; \underline{\mathbf{\div 6}} = 6y \; \underline{\mathbf{\div 6}}$

$-3342.283333 = y$

$-3342.28 = y$

Rounding

$75.67\underline{3}$	$75.67\underline{5}67$
75.67	75.68

4 or less, round down

5 or greater, round up

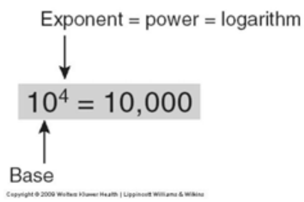
Exponents

An <u>exponent</u> represents the number of times a <u>base</u> is multiplied by itself.

$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

Exponents (cont.)

An exponent is also called a logarithm (log) or a power.



Exponents (cont.)

For base 10, with a <u>positive</u> whole number log, the number of zeros is equal to the log.

$10^0 = 1$

$10^1 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1000$

Exponents (cont.)

For base 10, with a <u>negative</u> whole number log, the number of decimal places is equal to the log.

$10^{-1} = 0.1$

$10^{-2} = 0.01$

$10^{-3} = 0.001$

$10^{-4} = 0.0001$

Exponents (cont.)

Use a calculator or log table if the log is not a whole number.

$10^{6.8} = 6309573.45$

Exponents (cont.)

Log expressions are written two ways:

$\text{Log}_{10}\ 1{,}268 = x$

$10^x = 1{,}268$

$x = 3.10$

Antilogs

When you raise a base to a log you get an <u>antilog</u>.

Antilog

Logarithm (exponent, power)

Base

$10^4 = 10{,}000$

Antilogs (cont.)

Antilog expressions are written two ways:

$$\text{Antilog}_{10}\ 3.2 = x$$

$$10^{3.2} = x$$

$$x = 1584.89$$

Scientific notation

Scientific notation is a way of representing very large or very small numbers in a condensed form.

$$a \times 10^{n}$$

$$1 \leq a < 10$$

n is an exponent

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Example

865,000,000,000

$$8.65 \times 10^{11}$$

The decimal moved 11 spaces to the left.

This number becomes the log.

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Example

0.00000000247

$$2.47 \times 10^{-9}$$

The decimal moved 9 spaces to the right.

Notice the log is negative in this case.

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Scientific notation (cont.)

Various calculator displays of 2.47×10^{-9}

2.47×10^{-9}

2.47 E -9

2.47 -9

2.47 ^ -9

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Converting from scientific to standard notation

$$6.54 \times 10^{7}$$

65400000.

Decimal to the right for positive logs

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Converting from scientific to standard notation (cont.)

4.35×10^{-5}

.0000435

Decimal to the left for negative logs

Dividing numbers in scientific notation

$$\frac{a \times b}{c \times d} = \frac{a}{c} \times \frac{b}{d}$$

Example

$$\frac{2.49 \times 10^{11}}{3.29 \times 10^{12}} = \frac{2.49}{3.29} \times \frac{10^{11}}{10^{12}} = 0.757 \times 10^{-1}$$

Example (cont.)

Notice what happened to this part of the expression.

$$\frac{10^{11}}{10^{12}} = 10^{-1}$$

Two basic log rules

a $\frac{x^a}{x^b} = x^{a-b}$

b $x^a \times x^b = x^{a+b}$

Example (cont.)

The answer:

0.757×10^{-1}

Examine the first number. Notice it does not follow the rule: $1 \leq a < 10$.

Example (cont.)

$$0.757 \times 10^{-1}$$

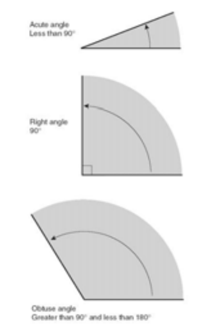
$$7.57 \times 10^{-2}$$

For conversion, the first number must become larger so the second number must become smaller.

Geometry

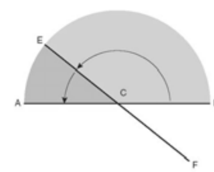
- Plane geometry
  - Two-dimensional figures
- Solid geometry
  - Three-dimensional objects

Angles



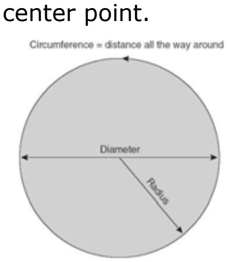
Angles (cont.)

- Line AB is intersected by line EF at point C.
- Angles above line AB are <ACE and <BCE.



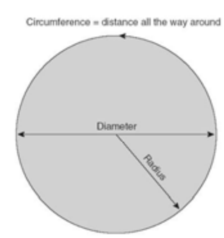
Circle

A <u>circle</u> is a curved line in which every point on it is the same distance away from the center point.



Circle (cont.)

The <u>radius</u> (r) is a line drawn from the central point to a point on the circle.

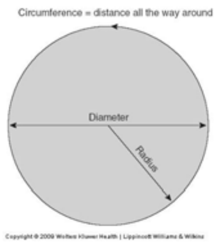


Circle (cont.)

The <u>circumference</u> (l) is the length of the entire circle.

$l = 2\pi r$

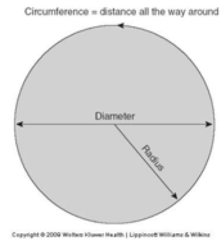
$\pi \approx 3.14$



Circle (cont.)

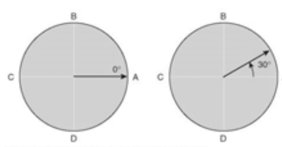
A straight line between two points that passes through the center is called the <u>diameter</u> (d).

$d = 2r$



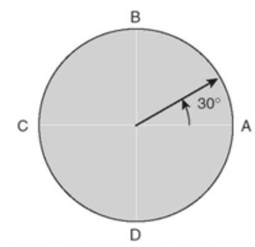
Circle (cont.)

- Notice a radius at point A.
- When the radius has rotated from point A to a new position, it has moved by a certain angle.



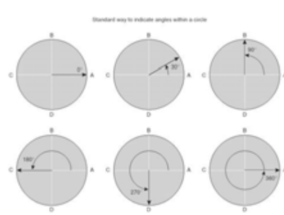
Circle (cont.)

- This angle is called <u>theta</u> and is written with the symbol θ.
- The angle is measured in degrees (º).



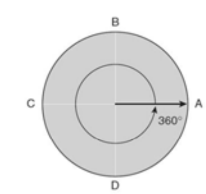
Circle (cont.)

Imagine the radius travels counter-clockwise around the entire circle like the hands of a clock traveling backwards.



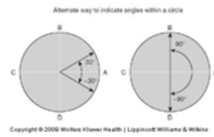
Circle (cont.)

One full rotation around the circle corresponds to 360º.



Circle (cont.)

- Sometimes the circle is divided into two halves:
  - 0° to 180º in one ½
  - 0° to -180° in the other ½



Circle (cont.)

- The other way to measure angles is with a unit called a <u>radian</u> (rad).
- The whole circle (360º) has 2π radians.

$$1\,\text{rad} = \frac{360^\circ}{2\pi} \approx 57.3^\circ$$

$$1^\circ = \frac{2\pi}{360^\circ} \approx 0.0175 rad$$

Circle (cont.)

- To convert from degrees to radians:

$$x(rad) = y^\circ \times \frac{\pi}{180^\circ}$$

- To convert from radians to degrees:

$$x^\circ = y(rad) \times \frac{180^\circ}{\pi}$$

Example

Convert 65º into radians

$$x(rad) = y^\circ \times \frac{\pi}{180^\circ}$$

$$x(rad) = 65^\circ \times \frac{\pi}{180^\circ}$$

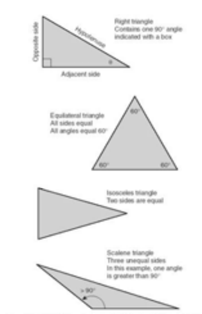
$$x(rad) = 65^\circ \times 0.01745$$

$$x(rad) = 1.13$$

65º = 1.13 radians

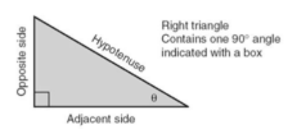
Trigonometry

Trigonometry is the branch of mathematics concerned with the study of triangles.



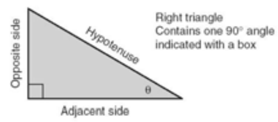
Right triangle

- A <u>right</u> triangle contains a 90º angle.
- The hypotenuse is the longest side of the right triangle.



Right triangle (cont.)

- The adjacent side is adjacent to the hypotenuse and forms the angle θ with the hypotenuse.
- The opposite side is the side opposite the angle θ.



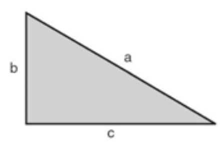
Pythagorean Theorem

The sum of the squares of the two shorter sides of a right triangle is equal to the square of the hypotenuse.

Pythagorean Theorem (cont.)

$$a^2 = b^2 + c^2$$

$$a = \sqrt{b^2 + c^2}$$



Example

If the two shorter sides of a triangle are 5.4 cm and 6.3 cm, what is the length of the hypotenuse?

$$a = \sqrt{b^2 + c^2}\,[cm]$$

$$a = \sqrt{5.4^2 + 6.3^2}\,[cm]$$

$$a = 8.30cm$$

Trigonometric functions

Trigonometric functions result when you divide the length of any one side of a right triangle by the length of another side.

Common trigonometric functions

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

Example

What is the sine of a 36º angle?

Use the sine key from a scientific calculator.

sine 36º= .59

Coordinate systems

Coordinates are distances or angles that uniquely identify the position of specific points in space in reference to a certain central point called the origin.

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The 2 coordinate systems

- Cartesian coordinates
  - (x, y)
  - (x, y, z)
- Polar coordinates
  - (r, θ)
  - (r, θ, φ)

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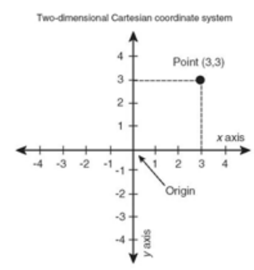
2-dimensional Cartesian Coordinates (x, y)



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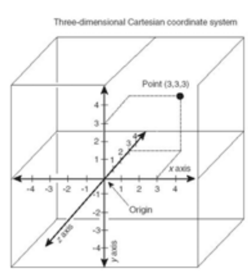
3-dimensional Cartesian Coordinates (x, y, z)



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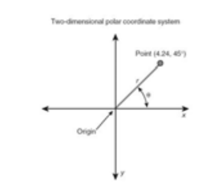
2-dimensional polar coordinates (r, θ)

- r is the distance between the point and the origin.
- θ is the angle between the x-axis and the line to the point.



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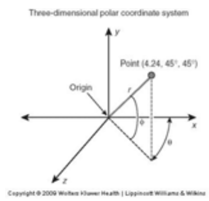
3-dimensional polar coordinates
(r, θ, φ)

The angle φ (phi) represents the direction in the vertical plane.



Polar to Cartesian conversion

x = r cos θ

y = r sin θ

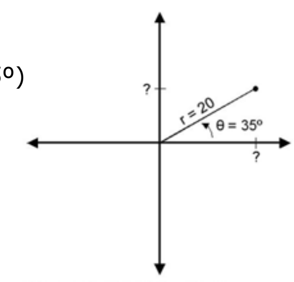
Cartesian to polar conversion

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(\frac{y}{x})$$

Example

Convert the polar coordinates (20, 35º) into Cartesian Coordinates.



Example (cont.)

First, determine the x coordinate

x = r cos θ

x = 20 cos (35º)

x = 20 × 0.81915

x = 16.38

Example (cont.)

Next, determine the y coordinate

y = r sin θ

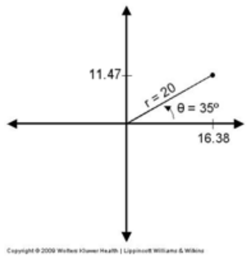
y = 20 sin (35º)

y = 20 × 0.57358

y = 11.47

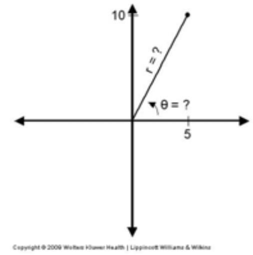
Example (cont.)

This graph shows the relationship between the polar coordinates and the Cartesian coordinates.



Example

Convert the Cartesian coordinates (5, 10) into polar coordinates.



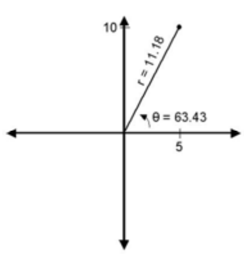
Example (cont.)

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{5^2 + 10^2}$$
$$r = \sqrt{125}$$
$$r = 11.18$$

Example (cont.)

$$\theta = \tan^{-1}(\frac{y}{x})$$
$$\theta = \tan^{-1}\left(\frac{10}{5}\right)$$
$$\theta = \tan^{-1} 2$$
$$\theta = 63.43^\circ$$

Example (cont.)



Functions

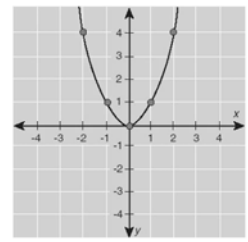
Cartesian and polar coordinates are usually used to describe a series of points that form a line.

Functions (cont.)

- The relationship between the coordinates that form a line is called a function.
- A <u>function</u> is an equation that shows the relationship between two sets of numbers.

The function $y = x^2$

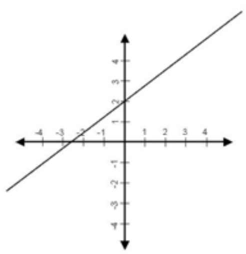
For every value of x, the value of y can be determined.



The function y =.82x + 2

Straight line functions have this form:

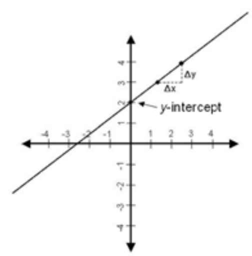
y = mx + b



The function y =.82x + 2

- b is the y-intercept
  - Here b = 2
- m is the slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = .82$$



Mathematics Homework

ANSWER KEY

1. Simplify $x = (56 + 18 \times 7) - 22(0.5 * 17)$.

$$x = (56 + 18 \times 7) - 22(0.5 * 17)$$

$$x = (56 + 126) - 22(8.5)$$

$$x = 182 - 187$$

Answer: $x = -5$

2. $\lambda = \frac{s}{f}$ Solve for f where $\lambda = 0.78$ and $s = 340$.

$$\lambda = \frac{s}{f}$$

$$0.78 = \frac{340}{f}$$

$$0.78 \times f = 1 \times 340$$

$$f = \frac{340}{0.78}$$

Answer: $f = 435.9$

3. Simplify $x = 10^{-0.28}$.

$$x = 10^{-0.28}$$

Answer: $x = 0.52$

4. Simplify $x = \log_{10} 0.89$.

$$x = \log_{10} 0.89$$

Answer: $x = -0.05$

5. Simplify $x = \text{antilog}_{10}\ 1.68$.

$$x = \text{antilog}_{10}\ 1.68$$

Answer: $x = 47.86$

(Note: $10^{1.68} = 47.86$)

6. Convert to scientific notation 54,300,000.

Answer: 5.43×10^{7}

7. Convert to scientific notation 862.57×10^{-5}.

Answer: 8.6257×10^{-3}

8. Convert 78° into radians.

$$x \text{ (rad)} = y\ (^\circ) \times \frac{\pi}{180^\circ}$$

$$x \text{ (rad)} = 78^\circ \times \frac{\pi}{180^\circ}$$

Answer: $x = 1.36$ rad

9. Calculate the sine of the angle $\theta = 54°$.

Answer: $\sin 54° = 0.81$

10. Convert Cartesian coordinates (51, 17) to polar coordinates.

$$r = \sqrt{x^2 + y^2} = \sqrt{51^2 + 17^2} = 53.76$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{17}{51}\right) = 18.43^\circ$$

Answer: (53.76, 18.43°)